

**A list of topics for preparation for the exam on  
“Operator semigroups and evolution equations”.**

- (1) An Abstract Cauchy Problem. Definitions of a semigroup, an infinitesimal generator. Different notions of continuity of a semigroup. The well-posedness Theorem. Example of a non-semigroup solution of a Cauchy problem for an evolution equation.
- (2) Uniformly continuous semigroups. Isomorphism between uniformly continuous semigroups and bounded linear operators. Uniqueness of the semigroup generated by a bounded linear operator. Standard properties of uniformly continuous semigroups. Multiplication semigroup.
- (3) Strongly continuous semigroups. Norm estimate. Contraction semigroups. Standard properties of strongly continuous semigroups and their generators. Uniqueness of a strongly continuous semigroup generated by a given operator.
- (4) Examples of strongly continuous semigroups on different Banach spaces: multiplication semigroup, translation semigroup, heat semigroup. Core criterium.
- (5) Pseudo-differential operators and their symbols. Bochner + Levy-Khinchin Theorem. Continuous negative definite functions, examples. Convolution semigroups, their connection with continuous negative definite functions. Operator semigroups associated with convolution semigroups, their strong continuity and the view of generators. Feller semigroups.
- (6) Resolvent set and resolvent of a closed operator, their properties. Resolvent of a generator of a strongly continuous semigroup as the Laplace transform of the semigroup, norm estimate of the resolvent. The resolvent equation. Neumann-series representation of the resolvent. Resolvent of a generator of a semigroup on  $L^2$  associated to a convolution semigroup.
- (7) Dissipative operators, their properties. Examples. Yosida approximations of a closed densely defined dissipative operator, their properties, examples.
- (8) The Hille-Yosida Theorem in the form of Lumer-Phillips. Steps of the proof. General version of the Hille-Yosida Theorem.
- (9) Relations between groups and semigroups, conditions of possibility to embed a strongly continuous semigroup into a strongly continuous group. The Hille-Yosida Theorem for groups. Adjoint operator. Generation of a strongly continuous contraction semigroup by a dissipative operator whose adjoint is also dissipative.
- (10) Adjoint semigroup. Strong continuity of the adjoint semigroup of a strongly continuous semigroup in the case of a reflexive Banach space. The Stone Theorem, steps of the proof.
- (11) Approximation and representation of semigroups based on the inversion of the Laplace transform. First and Second Trotter-Kato Approximation Theorems. The Chernoff Theorem, steps of the proof. The Post-Widder Inversion Formula. The Lie-Trotter Formula.